Week 15

Sunday, April 30, 2023 11:50 AM

In Week 14 slide pack:

Find Arc Length in Polar

$$\boxed{L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta}$$

ex. find arc length of $r = 1 + \sin\theta$

$$r^2 = (1+\sin\theta)(1+\sin\theta)$$
$$= (1 + 2\sin\theta + \sin^2\theta)$$
$$\left(\frac{dr}{d\theta}\right)^2 = (\cos\theta)^2$$

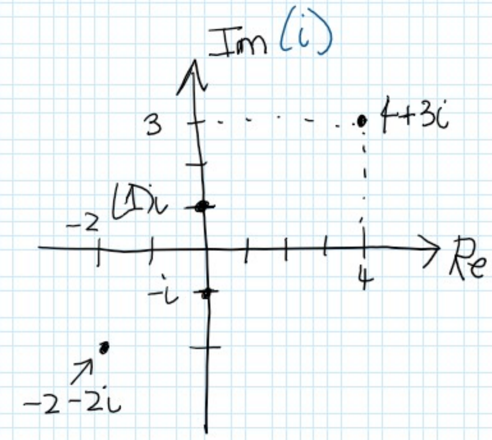
$$L = \int_0^{2\pi} \sqrt{1 + 2\sin\theta + \underbrace{\sin^2\theta + \cos^2\theta}_{1}}\, d\theta$$
$$= 2\int_0^{\pi} \sqrt{2 + 2\sin\theta}\, d\theta$$

use integral calculator

$$= \boxed{8}$$

Week 15 Appendix I Complex Numbers

so far, used Cartesian plane
polar plane



for complex numbers, use Argand plane
format $a + bi$ as ordered pair (a, b)

real component (a), imaginary component (bi)

plot points: $i \longleftrightarrow 0 + 1i$
$-i \longleftrightarrow (0, -1)$
$(4, 3) \longleftrightarrow 4 + 3i$
$-2 - 2i$

$$\boxed{\begin{aligned} i &= \sqrt{-1} \\ i^2 &= -1 \end{aligned}}$$

Arithmetic Operations w/ Complex Numbers

for addition/subtraction of complex numbers, treat real and imaginary components as "like" terms

ex. $(1 - i) + (4 + 7i)$
$= 1 - i + 4 + 7i$
$= \boxed{5 + 6i}$

ex. $(-1 + 3i)(2 - 5i)$
$= -2 + 5i + 6i - 15\underbrace{i^2}_{-1}$
$= -2 + 5i + 6i + 15$
$= \boxed{13 + 11i}$ — $a + bi$ format

Do: FOIL and combine like terms

ex. $\frac{-1+3i}{2-5i} \cdot \frac{2+5i}{2+5i} = \frac{-2-5i+6i-15}{4+25}$

no i in denom.

$= \frac{-17 + i}{29}$

$\downarrow a + bi$

$$\boxed{-\frac{17}{29} + \frac{1}{29}i}$$

$$\frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

use complex conjugate to cancel i from denominator:

$$a + bi \xleftrightarrow[\text{conjugate}]{\text{complex}} a - bi$$

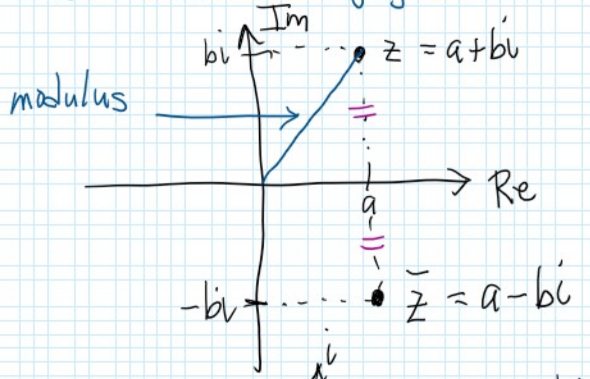
24 24

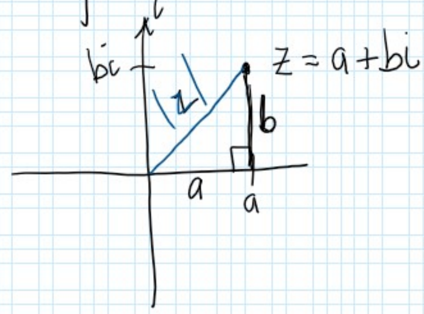
$a + bi \xleftrightarrow[\text{conjugate}]{\text{complex}} a - bi$

let $z = a + bi$ $\qquad w = c + di$

$\bar{z} = a - bi$ $\qquad \bar{w} = c - di$

z's complex conjugate





Other Conjugate Properties

$\overline{zw}$ ← conjugate of product $= \bar{z}\,\bar{w}$

complex # . complex #

$\overline{z + w} = \bar{z} + \bar{w}$

next significant item in this scenario!

modulus := distance of z from origin

$|z| = \sqrt{a^2 + b^2}$

it follows that $z\bar{z} = |z|^2$

because: $(a+bi)(a-bi)$

$a^2 + b^2 = |z|^2$

"modulus of z squared"

Find Complex Roots

$i^2 = -1$

$\sqrt{c}\,(i = \sqrt{-1}) \Rightarrow \sqrt{c}\,i = \sqrt{c}\sqrt{-1}$

$\sqrt{c} \cdot i = \sqrt{-c}$

ex. find the roots of $x^2 + x + 1 = 0$

use Quadratic Formula: $x = \dfrac{-1 \pm \sqrt{1 - 4(1)(1)}}{2(1)}$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-1 \pm \sqrt{1-4}}{2}$

$= \dfrac{-1 \pm \sqrt{-3}}{2}$

$\sqrt{-3} = \sqrt{3(-1)}$

$= \sqrt{3}\sqrt{-1}$

i

$= \dfrac{-1 \pm \sqrt{3}\sqrt{-1}}{2}$

$x = \dfrac{-1}{2} \pm \dfrac{\sqrt{3}\,i}{2}$

$a \pm bi$

$x_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$ $\qquad x_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$